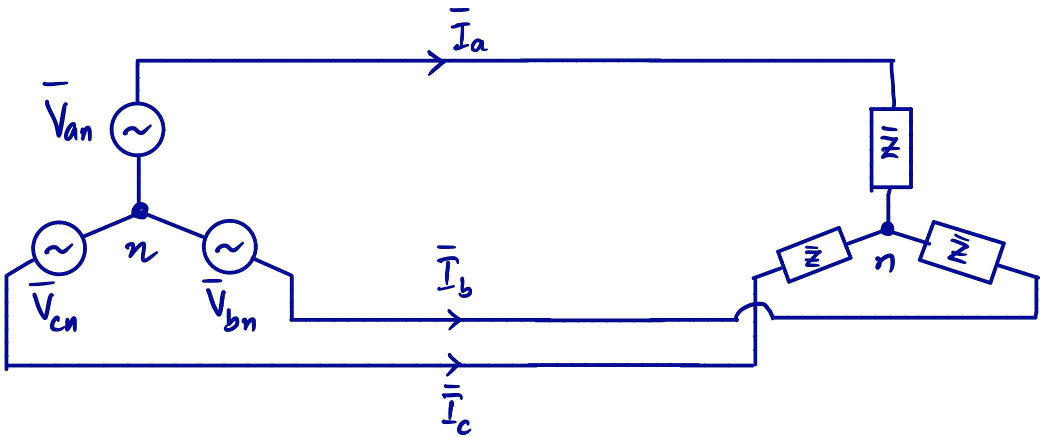
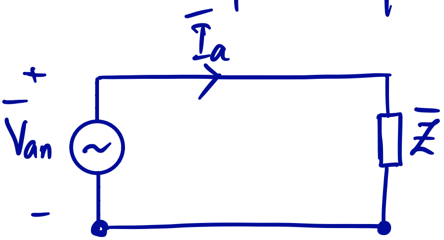
Per-phase equivalents:

- Y-source + Y-load:



Its per-phase equivalent is given by

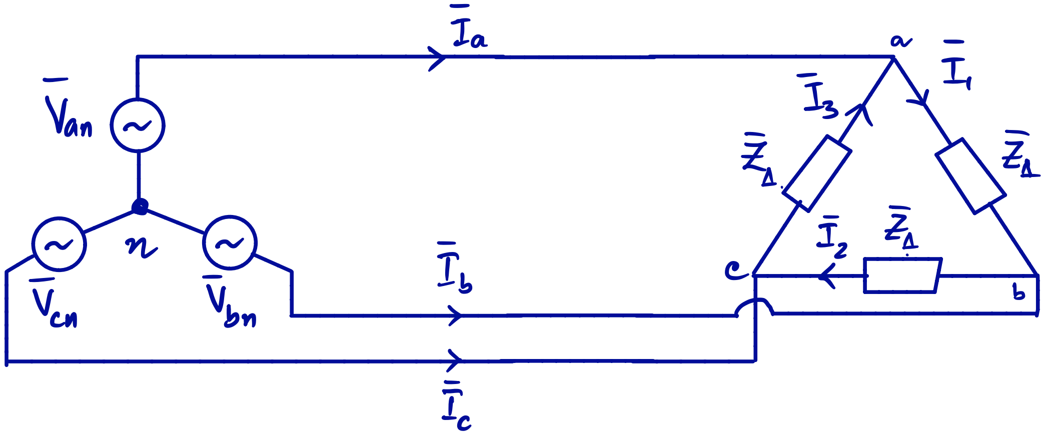


The single phase description has all information to compute all 3ϕ quantities.

For example, $\bar{I}_a = \dfrac{\bar{V}_{an}}{\bar{Z}}$, and you can calculate

$\bar{I}_b = \bar{I}_a \angle -120^\circ$, $\bar{I}_c = \bar{I}_a \angle +120^\circ$, etc.

- Y-source + Δ load:



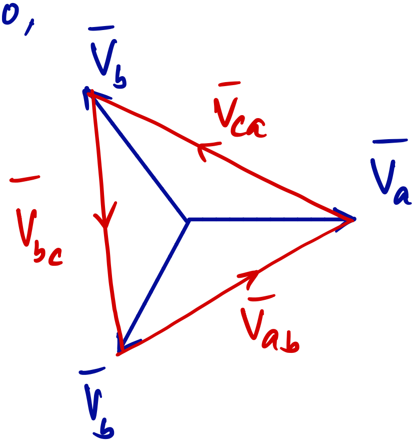
Let us transform the Δ-load to an equivalent Y-load. To simplify,

Assume that $\bar{V}_{an} = V_0 \angle 0$,

$\Rightarrow \bar{V}_{ab} = \sqrt{3}\, V_0 \angle 30^\circ$,

$\bar{V}_{bc} = \sqrt{3}\, V_0 \angle -90^\circ$,

$\bar{V}_{ca} = \sqrt{3}\, V_0 \angle 150^\circ$.

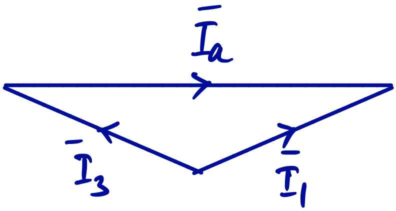


$$\bar{I}_1 = \frac{\bar{V}_{ab}}{\bar{Z}_\Delta} = \frac{\sqrt{3}\, V_0 \angle 30^\circ}{\bar{Z}_\Delta},$$

$$\bar{I}_2 = \frac{\bar{V}_{bc}}{\bar{Z}_\Delta} = \frac{\sqrt{3}\, V_0}{\bar{Z}_\Delta} \angle -90^\circ,$$

$$\bar{I}_3 = \frac{\bar{V}_{ca}}{\bar{Z}_\Delta} = \frac{\sqrt{3}\, V_0}{\bar{Z}_\Delta} \angle 150^\circ.$$

$$\therefore \bar{I}_a = \bar{I}_1 - \bar{I}_3 = \frac{\sqrt{3}\, V_0}{\bar{Z}_\Delta} \angle 30^\circ - \frac{\sqrt{3}\, V_0}{\bar{Z}_\Delta} \angle 150^\circ$$

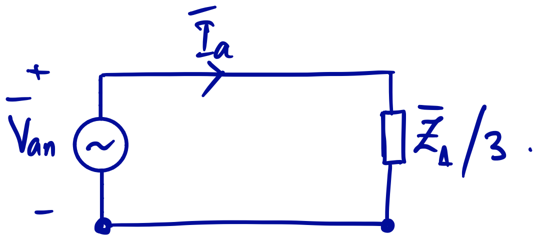


$$= \frac{1}{\bar{Z}_\Delta}\left[\sqrt{3}\, V_0 \angle 30^\circ - \sqrt{3}\, V_0 \angle 150^\circ\right]$$

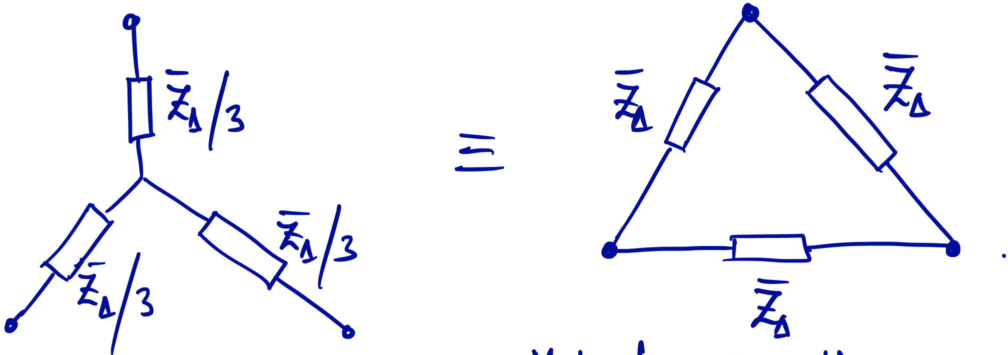
$$= \frac{1}{\bar{Z}_\Delta} \cdot \sqrt{3}\, V_0 \cdot \sqrt{3} \cdot \angle 0^\circ$$

$$= \frac{V_0}{\bar{Z}_\Delta / 3}.$$

$\therefore$ $\bar{V}_{an} = V_o \angle 0°$ drives a current of $\bar{I}_a = \dfrac{V_o}{(\bar{Z}_\Delta/3)}$ through the a-phase line, that suggests a per-phase equivalent circuit given by



Then, the load is the per-phase equivalent of a Y-connected load with $\bar{Z}_Y = \bar{Z}_\Delta/3$, i.e.,



... Y-Δ transformation.

Example #1

Consider a Δ-connected load with $V_L = 345$ kV, drawing 750 MVA at a pf of 0.8 lagging.

Compute:

1. Line and phase currents.
2. Real & reactive power drawn per phase.
3. Per phase impedance.

Solution: 1. $|\bar{S}_{3\phi}| = 750$ MVA

$$\Rightarrow \sqrt{3}\, V_L I_L = 750 \text{ MVA}$$

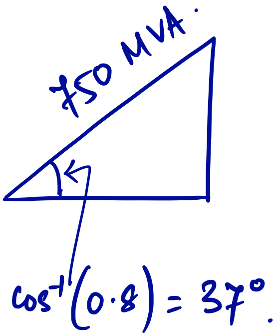
$$\Rightarrow I_L = \frac{750 \text{ MVA}}{\sqrt{3}\, V_L} = \frac{750 \text{ MVA}}{\sqrt{3} \cdot 345 \text{ kV}} = 1256 \text{ A}.$$

$$\Rightarrow I_\phi = \frac{I_L}{\sqrt{3}} = 725 \text{ A}.$$

2. $P_\phi = \frac{1}{3} \operatorname{Re}\{\bar{S}_{3\phi}\}$

Note the 1/3 in front of $\operatorname{Re}\{\bar{S}_{3\phi}\}$.

$Q_\phi = \frac{1}{3} \operatorname{Im}\{\bar{S}_{3\phi}\}$.



$$\therefore\ P_\phi = \frac{1}{3}\,\mathrm{Re}\{750 \angle 37°\}\ \text{MW}$$

$$= \frac{1}{3}\ 750 \times 0.8 \quad \text{MW}$$

$$= 200\ \text{MW}.$$

$$Q_\phi = \frac{1}{3}\,\mathrm{Im}\{750 \angle 37°\} \quad \text{MVAR}$$

$$= \frac{1}{3}\ 750 \times 0.6 \quad \text{MVAR}$$

$$= 150\ \text{MVAR}.$$

$$\bar{Z} = \underbrace{\frac{V_\phi}{I_\phi}}_{\text{why?}} \angle\theta = \frac{V_L}{I_\phi}\angle\theta = \left(\frac{345\ \text{kV}}{725\ \text{A}}\right)\angle 37°\ \Omega$$

$$= 476 \angle 37°\ \Omega.$$

Example #2.

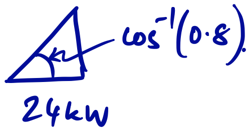
This means line-to-line voltage is 480 V.

Two 3ϕ loads are connected in parallel to a Y-connected 480 V source:

- Y-connected load 1: 24 kW at 0.8 pf lagging
- Δ-connected load 2: 30 kVA at 0.8 pf leading.

Find the line current phasors $\bar{I}_{1,a}$ and $\bar{I}_{2,a}$ for the two loads, total complex power $\bar{S}_T$ drawn by the loads, and the total line current drawn at the source. Let $\angle \bar{V}_{an} = 0$.

<u>Solution</u>: $\bar{S}_{3\phi,1} = \dfrac{24}{0.8} \angle 37^\circ$ kVA.



$$= 30 \angle 37^\circ \text{ kVA}.$$

$$\bar{S}_{3\phi,2} = 30 \angle -37^\circ \text{ kVA}.$$

$$\therefore \bar{S}_T = \bar{S}_{3\phi,1} + \bar{S}_{3\phi,2} = 48 \text{ kW}.$$

$V_L = 480$ V for both loads.

$$\Rightarrow |\bar{I}_{1,a}| = \frac{|\bar{S}_{3\phi,1}|}{\sqrt{3}\,V_L} = \frac{30\times10^3}{\sqrt{3}\cdot480}\text{ A} = 36.1\text{ A}.$$

$\Rightarrow \angle\bar{I}_{1,a} = -37°$. Why? Current lags voltage.

$$\Rightarrow |\bar{I}_{2,a}| = \frac{|\bar{S}_{3\phi,2}|}{\sqrt{3}\,V_L} = \frac{30\times10^3}{\sqrt{3}\cdot480}\text{ A} = 36.1\text{ A}.$$

$\Rightarrow \angle\bar{I}_{2,a} = +37°$.

$\therefore\ \bar{I}_{1,a} = 36.1\angle{-37°}$ A,

$\quad\ \bar{I}_{2,a} = 36.1\angle{+37°}$ A.

Total line current $= \bar{I}_{1,a} + \bar{I}_{2,a}$

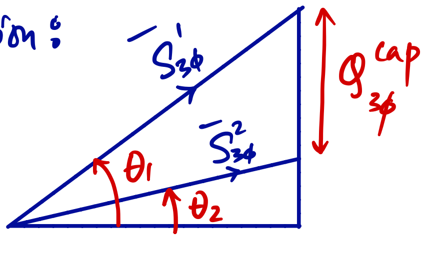
$$= (36.1\angle{-37°} + 36.1\angle{+37°})\text{A}$$

$$= 57.7\text{ A}.$$

Example # 3

An industrial plant consists of several 3ϕ induction motors and draws 300 kW at 0.6 pf lagging. How much per-phase reactive power should be injected from a 3ϕ capacitor bank in parallel to the plant to improve the net power factor to 0.9 lagging.

Solution:



$\theta_1 = \cos^{-1}(0.6) = 53.1^\circ$

$\theta_2 = \cos^{-1}(0.9) = 25.8^\circ$

Here, $\bar{S}^{2}_{3\phi}$ and $\bar{S}^{1}_{3\phi}$ are the complex powers with and without the capacitor banks.

$$\therefore Q^{cap}_{3\phi}\left(\text{injected by cap banks}\right) = 300\text{ kVAR}\left(\tan 53.1^\circ - \tan 25.8^\circ\right)$$

$$= 254.7 \text{ kVAR}$$